# MALMQUIST INDEX FOR GERMAN ENERGY NETWORK REGULATION

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# BACKGROUND

- German regulator “Bundesnetzagentur” is currently preparing the 3rd regulatory period for energy networks (2019-2023 for electricity)
- BNetzA needs to determine the general productivity factor ($X_{GEN}$) which is one part of the price adjustment formula known as “**RPI-X**”
- But what exactly is “X” and how should it be determined?
  - The basic decision is between the “Törnquist index” (used previously) and the “Malmquist index”
  - Main advantage of the Malmquist index: **separates the “frontier shift” from “catch-up” effects**: only the first one is relevant for RPI-X regulation

Based on joint research project:

Jacobs University and Polynomics (2016). “*Die Ermittlung des technologischen Fortschritts anhand von Unternehmensdaten – Der Einsatz der Malmquist-Methode im deutschen Regulierungsrahmen*“

► Analysis of the Malmquist index as a means to calculate $X_{GEN}$

► Proposal of a “**TOTEX Malmquist index**” to overcome data limitations of the regulator

# RPI-X REGULATION IN GERMANY

- To mimic competition:
  - Decoupling of revenues from actual cost of regulated firms
  - Change in revenues should reflect change in efficient costs:

$$\Delta R = \Delta C = \Delta w - \Delta TFP$$

$\Delta w$: input price change

$\Delta TFP$: Change in total factor productivity

- Regulatory formula:

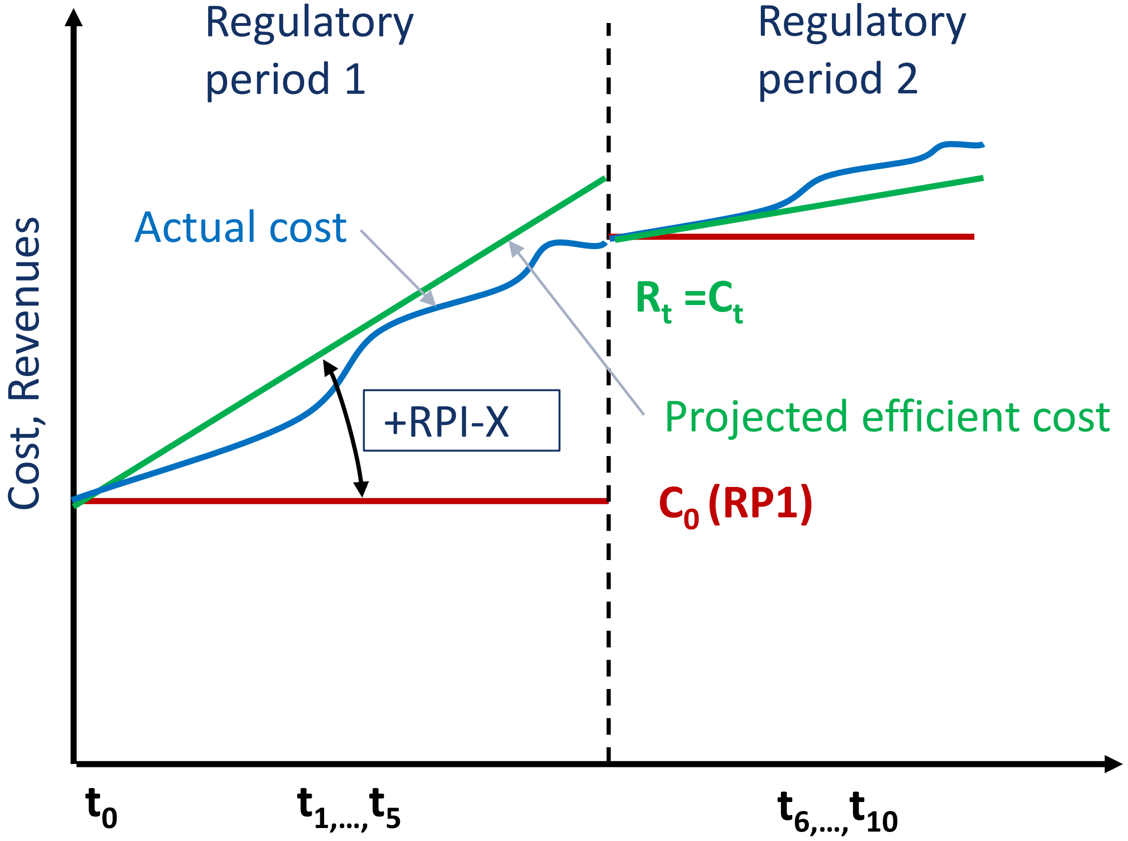
$$\Delta R = RPI_t - X_{GEN}$$
$$RPI_t = \Delta w^T - \Delta TFP^T$$

► $X_{GEN}$ used as correction term for residential price index (RPI):

$$X_{GEN} = (\Delta TFP^S - \Delta TFP^T) + (\Delta w^T - \Delta w^S)$$

$$\boxed{X_{GEN} = RPI_t - (\Delta w^S - \Delta TFP^S)}$$

► Problem: how to determine $\Delta \boldsymbol{w}$ and $\Delta \boldsymbol{TFP}$?



# OPTIONS TO DETERMINE ΔW AND ΔTFP

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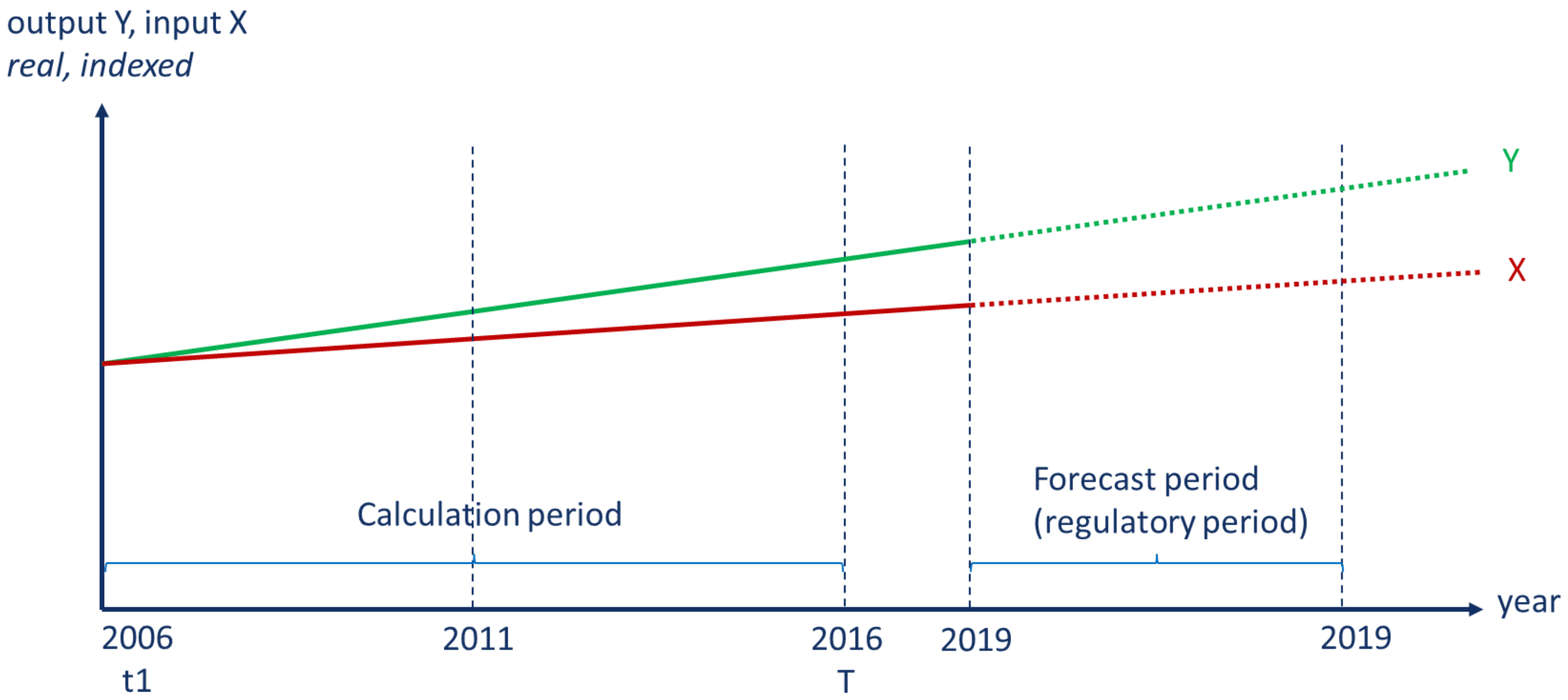
## 1) Törnquist Index

Calculations based on aggregate firm data




$$Törnquistindex(TI) = \frac{Outputindex}{Inputindex}$$

Case of one input and one output:

$$Outputindex = \sqrt[T-1]{\prod_{t=1}^{T-1} \frac{Y_{t+1}}{Y_t}} = \left(\frac{Y_T}{Y_1}\right)^{\frac{1}{T-1}}$$

$$Inputindex = \sqrt[T-1]{\prod_{t=1}^{T-1} \frac{X_{t+1}}{X_t}} = \left(\frac{X_T}{X_1}\right)^{\frac{1}{T-1}}$$

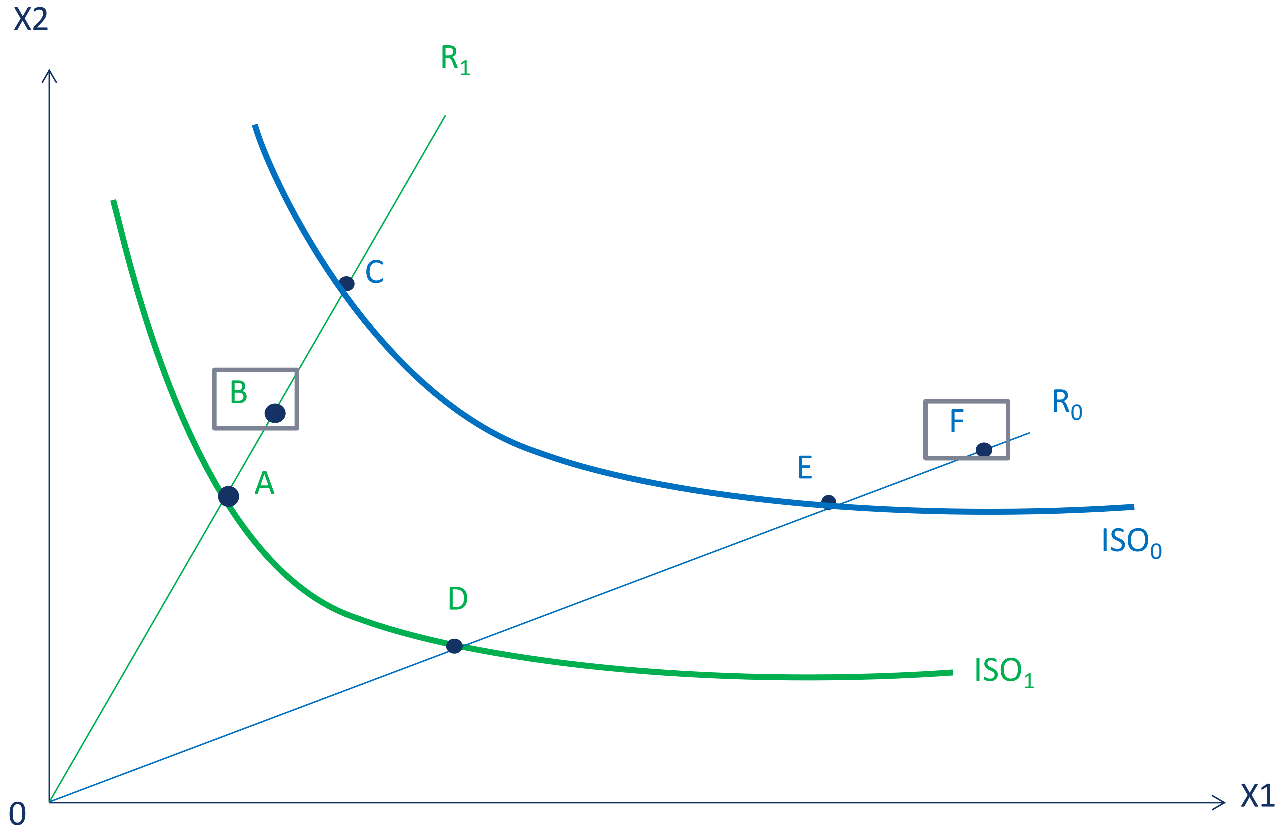
$$\boxed{TI = \left(\frac{Y_T/Y_1}{X_T/X_1}\right)^{\frac{1}{T-1}}}$$

Problem: no seperation between “catch-up” and “frontier shift”

# OPTIONS TO DETERMINE ΔW AND ΔTFP

## 2) Malmquist Index

Calculation based on benchmarking techniques (e.g. DEA)




Types

a) Production Malmquist Index (PMI)

[Färe et al., 1989]

► Only ΔTFP can be calculated;
Δw calculated separately

a) Cost Malmquist Index (CMI)

[Maniadakis, N., & Thanassoulis, 2004]

► Includes both ΔTFP and Δw, i.e. **efficient cost change**; input price and quantity data required

c) TOTEX Malmquist (TMI)

► PMI with total cost (TOTEX) as input given that regulator does not have separate price and input data

► Under which conditions does TMI lead to undistorted measure of the efficient cost change?

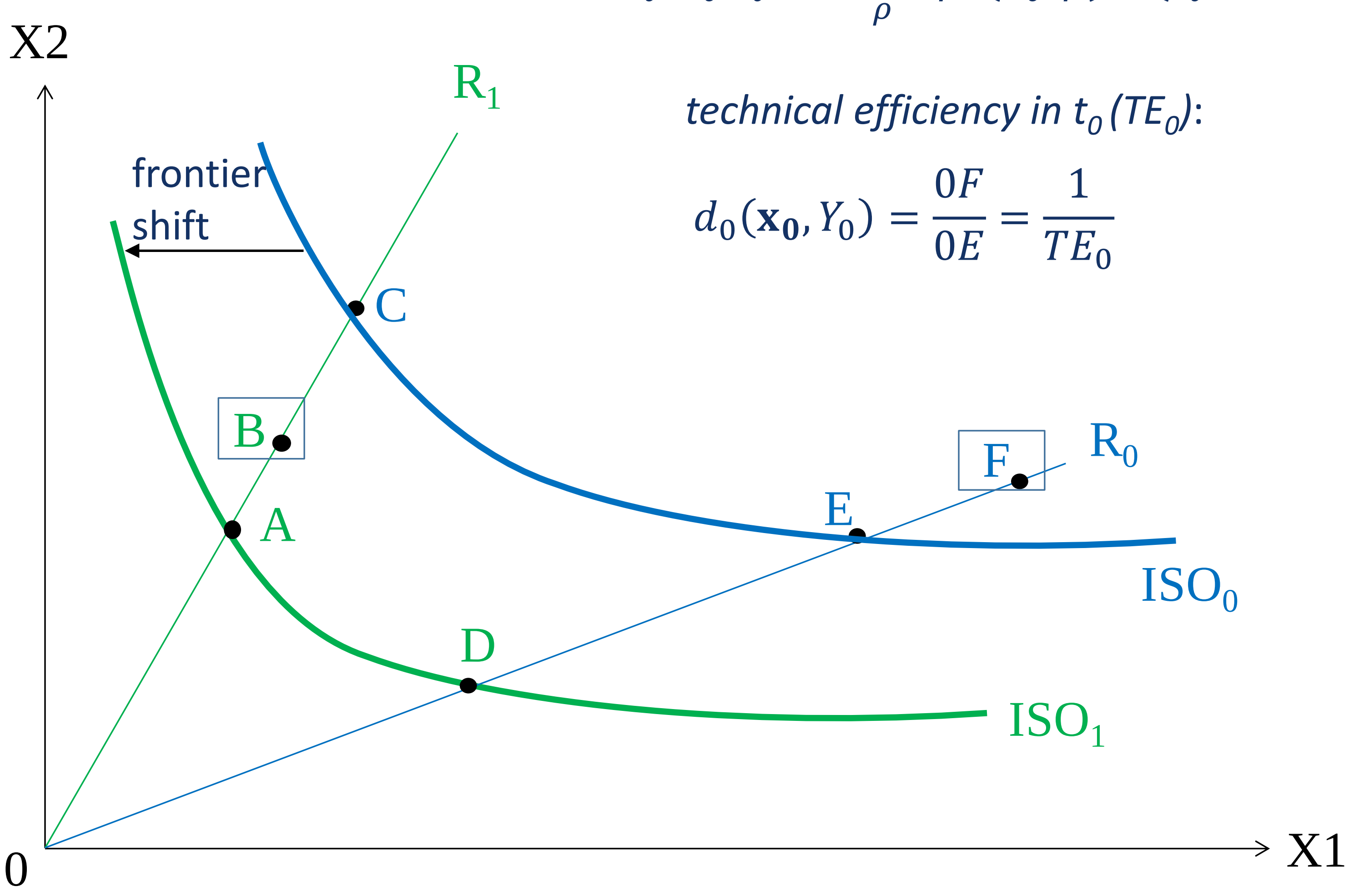
# PRODUCTION MALMQUIST INDEX (PMI)

Assume a firm producing at F in $t_0$ and B in $t_1$.

$$d_t(\mathbf{x_t}, Y_t) = \max_{\rho}\{\rho: (\mathbf{x_t}/\rho)\epsilon L(Y_t)\}$$

*technical efficiency in $t_0$ ($TE_0$):*

$$d_0(\mathbf{x_0}, Y_0) = \frac{0F}{0E} = \frac{1}{TE_0}$$




$$\text{PMI} = \left[\left(\frac{0B/0C}{0F/0E}\right)\cdot\left(\frac{0B/0A}{0F/0D}\right)\right]^{1/2}$$

$$= \underbrace{\frac{0B/0A}{0F/0E}}_{TEC}\cdot\underbrace{\left[\left(\frac{0B/0C}{0B/0A}\right)\cdot\left(\frac{0F/0E}{0F/0D}\right)\right]^{1/2}}_{TC}$$

With TEC: technical efficiency change (“catch-up“)

TC: technical change (“frontier shift”)

TC written in distance functions:

$$\boxed{\text{TC} = \left[\left(\frac{d_0(\mathbf{x_1}, Y_1)}{d_1(\mathbf{x_1}, Y_1)}\right)\cdot\left(\frac{d_0(\mathbf{x_0}, Y_0)}{d_1(\mathbf{x_0}, Y_0)}\right)\right]^{1/2}}$$

TC is the inverse of TFP:

$$\Delta\text{TC} = -\Delta\text{TFP}$$

But: PMI only regards physical inputs.

▶ Input price change **Δw** has to be determined separately

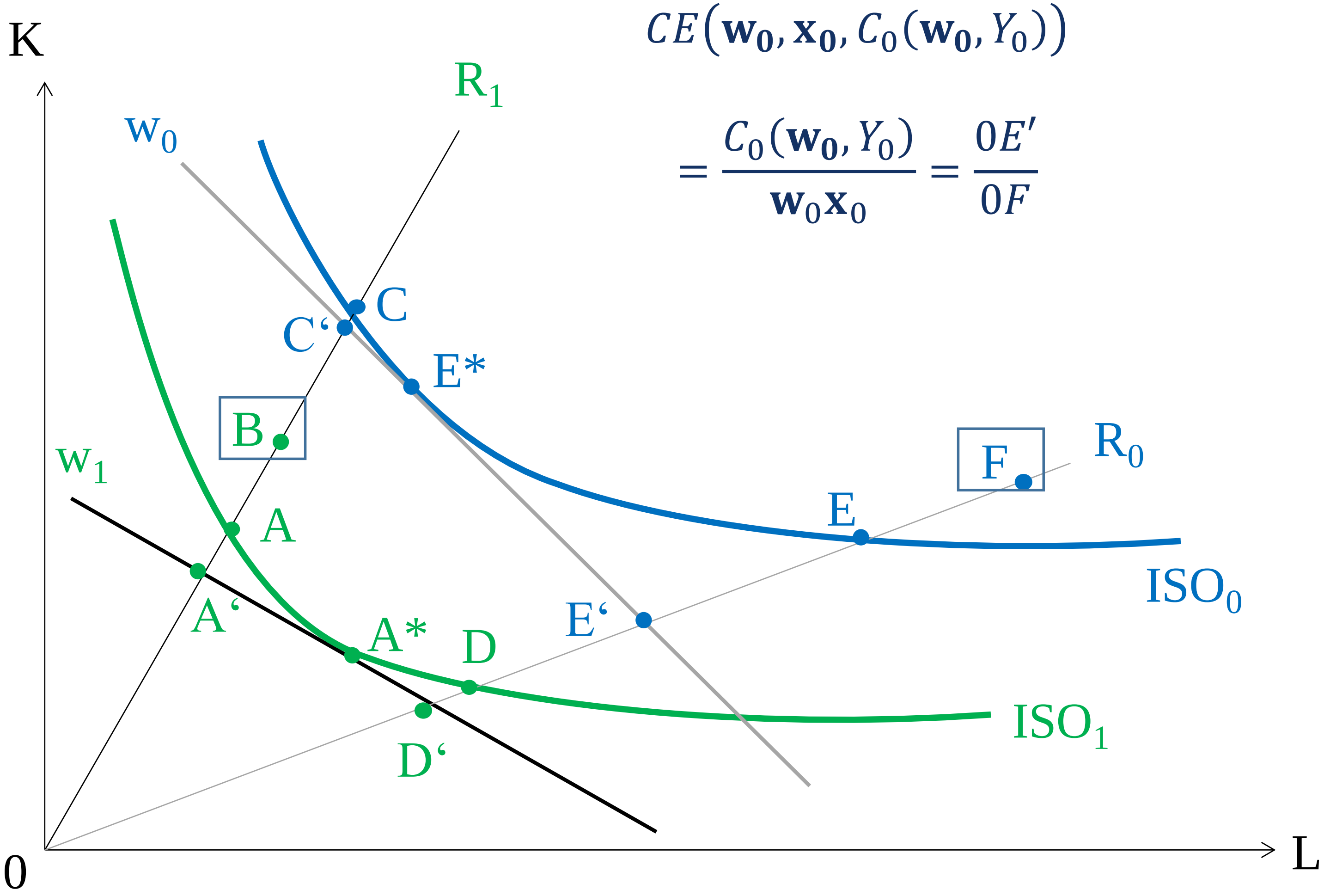
# COST MALMQUIST INDEX (CMI)



*Cost efficiency (CE)*:

$$CE\left(\mathbf{w_0}, \mathbf{x_0}, C_0(\mathbf{w_0}, Y_0)\right)$$

$$= \frac{C_0(\mathbf{w_0}, Y_0)}{\mathbf{w_0 x_0}} = \frac{0E'}{0F}$$




$$CMI = \left[\frac{C_0(\mathbf{w_0}, Y_0)/\mathbf{w_0 x_0}}{C_0(\mathbf{w_0}, Y_1)/\mathbf{w_0 x_1}} \times \frac{C_1(\mathbf{w_1}, Y_0)/\mathbf{w_1 x_0}}{C_1(\mathbf{w_1}, Y_1)/\mathbf{w_1 x_1}}\right]^{1/2}$$

$$= \underbrace{\left[\frac{C_0(\mathbf{w_0}, Y_0)/\mathbf{w_0 x_0}}{C_1(\mathbf{w_1}, Y_1)/\mathbf{w_1 x_1}}\right]}_{=OEC}$$

$$\times \underbrace{\left[\frac{C_1(\mathbf{w_1}, Y_1)/\mathbf{w_1} \boldsymbol{x_1}}{C_0(\mathbf{w_0}, Y_1)/\mathbf{w_0} \boldsymbol{x_1}} \times \frac{C_1(\mathbf{w_1}, Y_0)/\mathbf{w_1} \boldsymbol{x_0}}{C_0(\mathbf{w_0}, Y_0)/\mathbf{w_0} \boldsymbol{x_0}}\right]^{1/2}}_{=CTC}$$

With: OEC: overall efficiency change (“catch-up“)

CTC: cost technical change

CTC in efficiency terms:

$$CTC = \left[\frac{CE\left(\mathbf{w_1}, \mathbf{x_1}, C_1(\mathbf{w_1}, Y_1)\right)}{CE\left(\mathbf{w_0}, \mathbf{x_1}, C_0(\mathbf{w_0}, Y_0)\right)} \times \frac{CE\left(\mathbf{w_1}, \mathbf{x_0}, C_1(\mathbf{w_1}, Y_1)\right)}{CE\left(\mathbf{w_0}, \mathbf{x_0}, C_0(\mathbf{w_0}, Y_0)\right)}\right]^{1/2}$$

► CTC is the relevant term measuring the frontier shift with respect to costs.

<u>But CTC is not exactly what we need.</u>

# FROM COST TECHNICAL CHANGE (CTC) TO EFFICIENT COST CHANGE (ECC)

$$CTC = \left[\frac{C_1(\mathbf{w_1}, Y_1)/\mathbf{w_1}\boldsymbol{x_1}}{C_0(\mathbf{w_0}, Y_1)/\mathbf{w_0}\boldsymbol{x_1}} \times \frac{C_1(\mathbf{w_1}, Y_0)/\mathbf{w_1}\boldsymbol{x_0}}{C_0(\mathbf{w_0}, Y_0)/\mathbf{w_0}\boldsymbol{x_0}}\right]^{1/2} = \left[\frac{CE\big(\mathbf{w_1}, \mathbf{x_1}, C_1(\mathbf{w_1}, Y_1)\big)}{CE\big(\mathbf{w_0}, \mathbf{x_1}, C_0(\mathbf{w_0}, Y_0)\big)} \times \frac{CE\big(\mathbf{w_1}, \mathbf{x_0}, C_1(\mathbf{w_1}, Y_1)\big)}{CE\big(\mathbf{w_0}, \mathbf{x_0}, C_0(\mathbf{w_0}, Y_0)\big)}\right]^{1/2}$$

Further decomposing CTC yields:

$$CTC = \underbrace{\left[\frac{C_1(\mathbf{w_0}, Y_1)}{C_0(\mathbf{w_0}, Y_1)} \times \frac{C_1(\mathbf{w_0}, Y_0)}{C_0(\mathbf{w_0}, Y_0)}\right]^{1/2}}_{=TC} \times \underbrace{\left[\frac{C_1(\mathbf{w_1}, Y_1)}{C_1(\mathbf{w_0}, Y_1)} \times \frac{C_1(\mathbf{w_1}, Y_0)}{C_1(\mathbf{w_0}, Y_0)}\right]^{1/2}}_{\equiv w} \times \underbrace{\left[\frac{\boldsymbol{w_0 x_1}}{\boldsymbol{w_1 x_1}} \times \frac{\boldsymbol{w_0 x_0}}{\boldsymbol{w_1 x_0}}\right]^{1/2}}_{\equiv \text{bias}}$$

Combining the first two terms:

$$TC \times w = \left[\frac{C_1(\mathbf{w_1}, Y_1)}{C_0(\mathbf{w_0}, Y_1)} \times \frac{C_1(\mathbf{w_1}, Y_0)}{C_0(\mathbf{w_0}, Y_0)}\right]^{1/2} = ECC$$

$$\left[\frac{\boldsymbol{w_1 x_1^*}}{\boldsymbol{w_0 x_1^*}} \times \frac{\boldsymbol{w_1 x_0^*}}{\boldsymbol{w_0 x_0^*}}\right]^{1/2}$$ Fisher index

$$\Delta TC + \Delta w = \Delta w - \Delta TFP = \Delta ECC$$

- CTC can be calculated using DEA; ECC results after correcting CTC for the bias term
- But this requires input price and quantity data: e.g. for $CE\big(\mathbf{w_0}, \mathbf{x_1}, C_0(\mathbf{w_0}, Y_0)\big) = C_0(\mathbf{w_0}, Y_1)/\mathbf{w_0}\boldsymbol{x_1}$
- <u>What if regulators only have information on total cost (TOTEX)?</u>

# TOTEX MALMQUIST INDEX (TMI) AND TOTAL COST CHANGE (TCC)

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We define the TOTEX Malmquist as a production Malmquist where TOTEX is the only input

$$\hat{d}_t(C_t, Y_t) = \max_{\rho}\{\rho : (C_t/\rho) \epsilon F(Y_t)\}$$

$$\text{TMI} = \left[\left(\frac{\hat{d}_0(C_1,Y_1)}{\hat{d}_0(C_0,Y_0)}\right)\cdot\left(\frac{\hat{d}_1(C_1,Y_1)}{\hat{d}_1(C_0,Y_0)}\right)\right]^{1/2} = \frac{\hat{d}_1(C_1,Y_1)}{\hat{d}_0(C_0,Y_0)}\cdot\underbrace{\left[\left(\frac{\hat{d}_0(C_1,Y_1)}{\hat{d}_1(C_1,Y_1)}\right)\cdot\left(\frac{\hat{d}_0(C_0,Y_0)}{\hat{d}_1(C_0,Y_0)}\right)\right]^{1/2}}_{=TCC}$$

$$\text{TCC} = \left[\frac{C_1(\mathbf{w_1},Y_1)/\mathbf{w_1x_1}}{C_0(\mathbf{w_0},Y_0)/\mathbf{w_1x_1}}\times\frac{C_1(\mathbf{w_1},Y_1)/\mathbf{w_0x_0}}{C_0(\mathbf{w_0},Y_0)/\mathbf{w_0x_0}}\right]^{1/2} = \left[\left(\frac{\widehat{CE}_1(C_1,Y_1)}{\widehat{CE}_0(C_1,Y_1)}\right)\cdot\left(\frac{\widehat{CE}_1(C_0,Y_0)}{\widehat{CE}_0(C_0,Y_0)}\right)\right]^{1/2}$$

- Total cost change (TCC) can be used as an approximation for the efficient cost change ECC
- Calculating TCC with DEA only requires TOTEX information
- But TCC ignores information on changing input prices and allocative efficiency.

Under what conditions will TCC be an unbiased approximation of ECC?

# WHEN IS THE TOTEX MALMQUIST UNBIASED?

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- Given that the true efficient costs are not known, DEA calculates the cost change with regard to the most efficient firms in the sample: the „frontier firms“
- Results may be distorted, if frontier firms are technically and/or allocatively inefficient:

$$TCC = \left[\frac{C_1^F(\mathbf{w}_1, Y_1)/\mathbf{w}_1\mathbf{x}_1}{C_0^F(\mathbf{w}_0, Y_0)/\mathbf{w}_1\mathbf{x}_1} \times \frac{C_1^F(\mathbf{w}_1, Y_1)/\mathbf{w}_0\mathbf{x}_0}{C_0^F(\mathbf{w}_0, Y_0)/\mathbf{w}_0\mathbf{x}_0}\right]^{1/2} = \underbrace{\frac{C_1(\mathbf{w_1}, Y_1)}{C_0(\mathbf{w_0}, Y_0)}}_{ECC} \times \underbrace{\frac{C_0(\mathbf{w_0}, Y_0)/C_0^F(\mathbf{w}_0, Y_0)}{C_1(\mathbf{w_1}, Y_1)/C_1^F(\mathbf{w}_1, Y_1)}}_{OEC^F}$$

With $C_t^F$: observed cost frontier identified in the DEA

$\mathbf{w}_\mathrm{t}\mathbf{x}_{\boldsymbol{t}}^*(\mathbf{w}_\mathrm{t}, Y_t) = C_t^*$: unobserved, true cost frontier

- Condition for unbiased TCC: $\boxed{TCC = ECC \Leftrightarrow OEC^F = TEC^F \times AEC^F = 1}$

► TCC will be undistorted ($\mathrm{TCC} = ECC$) if either

a) Frontier firms are **technically and allocatively efficient,** or

b) Technical and allocative inefficiency **remains constant** (i.e. no economic catch-up over time)

# WHEN IS THE TOTEX MALMQUIST UNBIASED? (CONT'D)

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- The assumption of **technical efficiency** seems reasonable: given effective regulation, some firms will operate close to technical efficiency
- The problem of TCC is **allocative inefficiency**: requires instant adjustment of inputs to changes in factor prices.
  - Especially long-lived capital assets in infrastructure (eg. electricity networks) cannot be adjusted to price changes in the short term
  - TCC does not consider input prices, and therefore cannot identify allocative inefficiency.

However…

- What matters is the **change** in allocative inefficiency.
  - TCC may still be a good approximation, if efficiency catch-up is small (i.e. for relatively short observation periods).
- The relative advantage of the Malmquist index remains:
  - Törnquist index does not distinguish at all between catch- up and frontier shift.

# IMPLICATIONS FOR REGULATION

- The advantage of the Malmquist index (over Törnquist) is that it separates the frontier shift from catch-up effects: only the first one is relevant for RPI-X regulation.
- The cost Malmquist can be used to calculate both total factor productivity ($\Delta TFP$) and input price changes ($\Delta w$) in combination, which is exactly the efficient cost change ($\Delta ECC$) needed for the RPI-X formula:

$$X_{GEN} = RPI_t - (\Delta w^S - \Delta TFP^S) = RPI_t - \Delta ECC$$

- However, regulators often do not have input prices and quantity data, but only total cost (TOTEX)
- A TOTEX Malmquist only uses aggregated cost data and calculates the total cost change (TCC):

$$X_{GEN} = RPI_t - \Delta TCC$$

- Not considering relative price changes, TCC is only an unbiased approximation for ECC, if the frontier firms in the data sample are not subject to allocative and technical efficiency changes over time.
- The size of distortions is difficult to approximate; however, especially DEA is known to be sensitive to outliers in the data.

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# THANK YOU FOR YOUR ATTENTION.

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